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A General Equilibrium Quantification of the Impact of Fraunhofer on the German Economy

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[1]This study has been commissioned by Fraunhofer. I have been compensated to complete it. The analysis and conlcusions of the study are in no way affected by the contractual commitment I have. There are three critical pieces I have used to conduct this study. Two of them were completed before I initiated this project. These are a study by Deloitte for the European Commission on the estimation of ideas production functions in which I participated, and a study I conducted together with Georg Licht, Maikel Pellens, and Torben Schubert in which we estimated the micro-economic effect of Fraunhofer reserach contracts on firm performance. I am grateful to all the reserachers that participated in those studies as they have proven critical for this paper. The third critical piece for this study has been the framework I have developed which is largely original. All remaining errors are my own.

Abstract

I construct a general equilibrium model with endogenous development and adoption of technologies at the firm level. The model recognizes the dual role of Fraunhofer as an institution that conducts public R&D and that helps some firms close their technological gap with the frontier through research contracts. I use a calibrated version of the model to calculate the long-run contribution of FhG to the German economy. The key finding is that FhG leads to a long-run level of GDP in Germany 3.65% higher than if Fraunhofer did not existed. The magnitude of this effect is robust. Both the public R&D and research contracts contribute to this effect, though the effect of FhG research on GDP via the increase in public R&D is much greater than the effect of research contracts on technology adoption.

Keywords: Fraunhofer, technology adoption, innovation, reserach contracts, publis R&D, general equilibrium.

JEL Classification: E3, O3.

1 Motivation

Studies of the impact of innovation policy have largely focused on R&D subsidies (e.g., Almus and Czarnitzki, 2003, Bronzoni and Piselli, 2016) and the strength of the patent protection conferred to inventors (Budish, Roin and Williams, 2013, Williams, 2015). This list contrasts with the large diversity of tools regional, national and supra-national governments use to foster a wide range of innovation activities. In this paper, we contribute to closing the gap by exploring the aggregate impact of an under-researched institution created in Germany in the aftermath of WWII to restore innovation activity: The Fraunhofer Society (Fraunhofer).

Fraunhofer is a large, public, applied research organization. Currently, it employs around 25,000 scientists and engineers. Its budget represents 7.7% of German public R&D. Every year, it obtains around 500 new patents and engages in research contracts with around 6500 individual companies that help them solve technological problems for which the market does not provide easy-to-find solutions (Comin et al., 2016).

Before assessing the aggregate effect of Fraunhofer we need to resolve several methodological questions. An immediate concern is whether individual organizations have aggregate effects on the economy. In macroeconomic models, we typically assume that firms are atomistic, and therefore their individual actions do not have aggregate effects. In the other end of the spectrum, the government is big and its actions have aggregate effects. On that continuum, where does Fraunhofer fall? Is it atomistic, or does it have some mass, albeit less than the government?

A second issue we need to resolve is how to introduce Fraunhofer in a model of the German economy. The approach I follow is to consider the dual role of Fraunhofer as an agent that conducts public R&D that enhances the productivity of private efforts to conduct innovations, and as an institution that helps some firms close their individual gap with the world technology frontier through research contracts.

Finally, we need to define a conceptual framework to evaluate the impact of Fraunhofer. Since the core endogenous variables we are interested in are the size/growth of the economy and the amount and nature of innovation activity conducted in the economy, I will conduct my analysis in a semi-endogenous growth model, similar to Jones (1995). The model will allow for the possibility that the world technology frontier is expanded through the development of both domestic and international innovations. Consistent with Deloitte (2017), I differentiate between public and private R&D in the idea production function. This opens the first avenue for Fraunhofer to impact the economy, through the impact that Fraunhofer applied research activities have on the expansion of the world technology frontier.

Realistically, we recognize that firms need to adopt and implement new technologies before they can use them for production. The gradual adoption of new technologies creates a diffusion curve that defines the pace at which new technologies penetrate the economy. Incorporating adoption/diffusion into our framework is important for at least two reasons. First, omitting it would greatly distort the quantitative predictions of the model. Additionally, it opens the second channel by which Fraunhofer impacts the economy as research contracts, by bringing firms closer to the frontier, increase the rates of adoption of new technologies for the subset of firms that directly interact with Fraunhofer.

After developing the model, I calibrate it to match the German economy. Then, I conduct a counterfactual exercise by which I eliminate Fraunhofer and compute the equilibrium steady state of the economy without Fraunhofer. The difference in the variables of interest between the two steady states provides an exact measure of the long-run effect that Fraunhofer has on the German economy.

The main findings of the analysis are as follows:

1. Fraunhofer significantly contributes to output, productivity, the world technology frontier and the technology adoption level in Germany.

2. In the baseline, Fraunhofer increases the long-run level of output, wages and productivity by 3.65%

3. The long-run level of technology increases by 13.49%

4. The speed of technology diffusion increases by 0.36%

5. The results are robust to variations in the calibration of idea production function, and the average adoption rate in the economy.

This study borrows from various strands of the literature. First, the framework I develop is related to a number of papers that construct endogenous growth and endogenous adoption models (e.g., Romer, 1990, Jones, 1995, and Comin and Gertler, 2006).[1] There are two key differences with these frameworks. The first is that in our model each firm needs to adopt each technology. This feature implies that there is a smooth diffusion curve for each new technology, while, for example, in Comin and Gertler (2006), the diffusion of a specific technology is a binary state. The second difference is that we explicitly build in Fraunhofer's activities into the model as this is the first paper that investigates the contribution of Fraunhofer to the economy. A second strand of the literature mentioned above has studied the effects of standard innovation policies such as R&D subsidies or tax breaks, as well as features of the patent system (e.g., Bloom, Griffith and Van Reenen, 2002). In addition to using a richer model than these analyses, as mentioned above, we focus on the effects of a distinct institution. A study that is closely connected to this one is Comin et al. (2017) who explores the micro-economic effect of Fraunhofer research contracts on firm performance. Unlike our setting, this study cannot assess the general equilibrium effects of research contracts or the interaction between research contracts and overall innovation activity. Finally, to conduct our quantification, we build on the work by various papers that have estimated the values of some of the critical parameters in our model. These include, for example, Bottazzi and Peri (2008), Deloitte (2017) who estimate the ideas production function, Anzoategui et al. (2019), Comin and Hobijn (2010) who estimate the parameters in the adoption function, and Comin et al. (2019) who estimate the impact on firm's value added of interacting with Fraunhofer through research contracts.

[1] For example, the models used by various branches of the European Commission (e.g., DG Innovation and EcFin) to evaluate the impact of innovation policies in the economy QUEST III, and updates conducted by Deloitte, 2017) belong to this general class of models.

The remaining of the paper is organized as follows. Section 2 describes the analytic framework. Section 3 describes the calibration and conducts the counterfactual exercise. Section 4 concludes.

2 Model

We present two versions of the same model. For expositional clarity, we start with the version without Fraunhofer. Once the setting is presented, we explain how introducing Fraunhofer changes the model dynamics. We describe the equilibria in both versions and describe the steady state of the economy used in section 3 to conduct the counterfactual exercises.

2.1 Setting

The baseline model has a semi-endogenous growth structure (Jones, 1995) with limited knowledge spillovers that make necessary the continuous expansion of the size of the market (measured by population) to sustain growth in the long run. Accordingly, we assume that population grows at rate n, both domestically and worldwide. Innovations are developed both domestically and abroad, though we do not explicitly model the investment decisions of foreign innovators. Innovations are embodied in intermediate goods that are commercialized domestically.[2] Final output firms use labor and intermediate goods for production activities. As in Romer (1990), they enjoy productivity gains from the number of intermediate goods they use in production. However, unlike Romer (1990), each firm must adopt an intermediate good before using it. Success in adoption investments defines the subset of final output producers that can use a given intermediate good at a point in time. As time goes by, the number of producers that have a adopted a given intermediate good increases generating a diffusion curve. These adoption dynamics define the present discounted value of demand faced by intermediate goods producers and hence their incentives to invest in the development of new intermediate goods. As a result, the setting creates a connection between adoption and innovation.

Technology frontier and types of technologies

Z_t denotes the state of technology which maps into the measure of intermediate goods that have been invented. Intermediate goods can be developed domestically or abroad. We denote domestic intermediate goods by Z_{Dt} and foreign by Z_{Ft}. The non-rivalry of ideas implies that a given intermediate good will be developed only once, therefore,

$$Z_t = Z_{Dt} + Z_{Wt} \tag{1}$$

[2] In this regard, we do not model the role of exports on the incentives of German innovators which means that, other than for the fact that we allow the forntier to be enhanced also by foreign R&D, the model treats the economy as closed.

The laws of motion of Z_W and Z_D are:

$$Z_{Wt+1} - Z_{Wt} = \zeta_{Wt} Z_{Wt}^{\phi} \, Z_{Dt}^{\xi} S_{GW}^{\kappa} S_W^{\lambda_z} \tag{2}$$

$$Z_{Dt+1} - Z_{Dt} = \zeta_{Dt} Z_{Dt}^{\phi} Z_{Wt}^{\xi} S_{GD}^{\kappa} \, S_t^{\lambda_z} \tag{3}$$

Suppose population grows at rate n. This implies that in steady state, where the share of workers in R&D activities is constant, the growth rate of Z, Z_W and Z_D is $g = (\lambda_z + \kappa)n/(1-\phi-\xi)$.

Production

There is a continuum of final goods producers in the economy indexed by j. Each final good firm produces a differentiated output using the following production function

$$Y_{jt} = A_{jt}^{\phi-\alpha(\mu-1)} X_{jt}^{\alpha} L_{jt}^{1-\alpha} \tag{4}$$

$$X_{jt} = \left(\int_0^{A_{jt}} x_{ijt}^{1/\mu} di \right)^{\mu}, \text{ with } \mu > 1, \tag{5}$$

where X_{jt} denotes a composite intermediate good used by firm j, x_{ijt} is the number of units of intermediate good i demanded by producer j at time t, and A_{jt} is the stock of technologies it has adopted.

Let p_{it}^x denote the price of intermediate good i at time t, and MC_{jt} the marginal cost of producing output j. Cost minimization yields the following demand for intermediate goods from producer j :

$$x_{ijt} = \frac{MC_{jt} Y_{jt} \alpha}{P_{jt}^X} \left(\frac{P_{jt}^X}{p_{it}^x} \right)^{\frac{\mu}{\mu-1}} \tag{6}$$

It takes $\bar{a}$ units of labor to produce one unit of intermediate good. If we denote by $\bar{\mu}$ the markup charged by intermediate good producers, and by W_t the wage rate, then the price of intermediate goods (p_{it}^x), the price of the intermediate good composite for firm j (P_{jt}^X), and the marginal cost of production for firm j (MC_j) are given by:

$$p_{it}^x = \bar{\mu}\bar{a}W_t$$

$$P_{jt}^X = A_{jt}^{-(\mu-1)} p_{it}^x$$

$$MC_{jt} = A_{jt}^{-\phi} \left(\frac{\bar{\mu}\bar{a}}{\alpha} \right)^{\alpha} (1-\alpha)^{-(1-\alpha)} W_t \tag{7}$$

Aggregate output, Y_t, is competitively produced by combining the differentiated outputs as follows.

$$Y_t = \left(\int_0^1 Y_{jt}^{1/\theta} dj \right)^{\theta} \tag{8}$$

Cost minimization implies that the demand for final output j is

$$Y_{jt} = Y_t \left(P_{jt}^y \right)^{-\frac{\theta}{\theta-1}} \tag{9}$$

where the price of final output j, P^y_{jt}, is

$$P^y_{jt} = \bar{\theta} MC_{jt}. \tag{10}$$

Substituting (10) and (7) in (9), we obtain the following expression for the amount of output produced by the j^{th} differentiated producer:

$$Y_{jt} = Y_t \left[\bar{\theta} * A_{jt}^{-\phi} \left(\frac{\bar{\mu}\bar{a}}{\alpha} \right)^{\alpha} (1-\alpha)^{-(1-\alpha)} W_t \right]^{-\frac{\theta}{\theta-1}}$$

The demand by producer j of each intermediate good it has adopted is

$$x_{ijt} = A_{jt}^{-(\phi+1)+\frac{\phi\theta}{\theta-1}} Y_t W_t^{-\frac{\theta}{\theta-1}} \left(\frac{\alpha}{\bar{\mu}\bar{a}} \right) \Xi$$

where $\Xi \equiv \left[\frac{(1-\alpha)^{1-\alpha}}{\bar{\theta}^{\theta} \left(\frac{\bar{\mu}\bar{a}}{\alpha} \right)^{\alpha}} \right]^{\frac{1}{\theta-1}}$.[3]

Assumption: We impose the constraint that $\phi = \theta - 1$, to ensure that x_{ijt} does not depend on A_{jt}, and that profits are linear in A_{jt}.

Cost minimization yields the following demand for labor and the intermediate good composite:

$$\begin{aligned} L_{jt} &= (1-\alpha) \frac{MC_{jt} Y_{jt}}{W_t} \\ X_{jt} &= \alpha \frac{MC_{jt} Y_{jt}}{P^X_{jt}} \end{aligned}$$

The operating profits for the j^{th} final output producer are:

$$\begin{aligned} \pi_{jt} &= P^y_{jt} Y_{jt} \left(1 - \frac{1}{\bar{\theta}} \right) \\ &= Y_t \left(\frac{\bar{\theta}-1}{\bar{\theta}} \right) A_{jt}^{\frac{\phi}{\theta-1}} \left[\bar{\theta} \left(\frac{\bar{\mu}\bar{a}}{\alpha} \right)^{\alpha} (1-\alpha)^{-(1-\alpha)} W_t \right]^{-1/(\theta-1)} \\ &= Y_t W_t^{-\frac{1}{\theta-1}} \left(\frac{\bar{\theta}-1}{\bar{\theta}} \right) A_{jt}^{\frac{\phi}{\theta-1}} \bar{\theta} \Xi \end{aligned}$$

[3] Note that

$$\begin{aligned} \frac{MC_{jt}\alpha}{P^X_{jt}} &= A_{jt}^{-\phi+\alpha(\mu-1)} \left(\frac{P^X_{jt}}{\alpha} \right)^{-(1-\alpha)} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \\ &= A_{jt}^{(\mu-1)-\phi} \left(\frac{\bar{\mu}\bar{a}}{\alpha} \right)^{-(1-\alpha)} (1-\alpha)^{-(1-\alpha)}. \end{aligned}$$

Total demand for labor for final output and intermediate goods:

$$
\begin{aligned}
L_{yt} &= \int_0^1 L_{jt}dj = \frac{(1-\alpha)}{w_t\bar{\theta}}\int_0^1 P_{jt}^{y}Y_{jt}dj \\
&= \frac{(1-\alpha)}{\bar{\theta}}Y_tW_t^{-\frac{\theta}{\theta-1}}\left[\bar{\theta}\left(\frac{\bar{\mu}\bar{a}}{\alpha}\right)^{\alpha}(1-\alpha)^{-(1-\alpha)}\right]^{-1/(\theta-1)}\int_0^1 A_{jt}dj \\
&= (1-\alpha)Y_tW_t^{-\frac{\theta}{\theta-1}}\left[\frac{(1-\alpha)^{(1-\alpha)}}{\bar{\theta}^{\theta}(\bar{\mu}\bar{a}/\alpha)^{\alpha}}\right]^{1/(\theta-1)}\int_0^1 A_{jt}dj \\
&= (1-\alpha)Y_tW_t^{-\frac{\theta}{\theta-1}}\Xi\int_0^1 A_{jt}dj \\
L_{xt} &= \bar{a}\int_0^1 A_j x_{ijt}dj = Y_tW_t^{-\frac{\theta}{\theta-1}}\left(\frac{\alpha}{\bar{\mu}}\right)\Xi\int_0^1 A_{jt}dj
\end{aligned}
$$

Aggregating across firms, we can solve for the total demand faced by intermediate good producer i :

$$
x_{it} = \int_0^1 1(Ad_{i,j}) * x_{ijt}dj = Y_tW_t^{-\frac{\theta}{\theta-1}}\left(\frac{\alpha}{\bar{\mu}\bar{a}}\right)\Xi\int_0^1 1(Ad_{i,j})dj \tag{11}
$$

where $1(Ad_{i,j})$ is an indicator function that takes the value of 1 if producer j has adopted the technology i, and it is 0 otherwise.

$$
A_t = \int_0^1 A_{jt}dj \tag{12}
$$

as the average technology level in the economy.

Adoption

Company j adopts an intermediate good with probability

$$
\lambda(h_{j\tau}) = \lambda_0\left(A_th_{j\tau}\right)^{\rho_\lambda} \tag{13}
$$

where $\lambda(.)$ is increasing in the number of workers employed in adoption of technology τ, $h_{j\tau}$. The resulting law of motion for the stock of adopted technologies by firm j is:

$$
A_{jt+1} - A_{jt} = \lambda_{jt}\left(Z_t - A_{jt}\right) \tag{14}
$$

Let q_t^A denote the value for a given firm of having adopted a technology at time t. q_t^A can be defined by the following Bellman equation:

$$
q_t^A = \frac{\partial \pi_{jt}}{\partial A_{jt}} + E_t\left(\Gamma_{t,t+1}v_{t+1}^A\right) \tag{15}
$$

where $\Gamma_{t,t+1}$ is the stochastic discount factor between t and $t+1$, and

$$
\frac{\partial \pi_{jt}}{\partial A_{jt}} = Y_tW_t^{-\frac{1}{\theta-1}}\left(\frac{\bar{\theta}-1}{\bar{\theta}}\right)\bar{\theta}\Xi \tag{16}
$$

Because $\frac{\partial \pi_{jt}}{\partial A_{jt}}$ and $\Gamma_{t,t+1}$ are not firm-specific, the marginal value of an intermediate good, q_t^A, is the same for all final output producers. h_t is the solution to the following maximization problem, which satisfies the first order condition (17). Note that since q_t^A and $\Gamma_{t,t+1}$ are the same for all firms, so is h_t, and $\lambda_{jt} = \lambda_t$, $\forall j$.

$$\breve{q}_t^A = \max_{h_t} \lambda\left(h_t\right) E_t\left[\Gamma_{t,t+1} q_{t+1}^A\right] - W_t h_t$$

$$\lambda'(h_t) E_t\left[\Gamma_{t,t+1} q_{t+1}^A\right] = W_t \tag{17}$$

The total number of workers employed in adopting new technologies is

$$H_t = \int h_{jt}\left(Z_t - A_{jt}\right) dj = h_t (Z_t - A_t) \tag{18}$$

where $A_t = \int_0^1 A_{jt} dj$ is the average number of adopted technologies across companies.

Endogenous innovation

Let $v_{t+1,0}^A$ be the market value of a newly invented technology at time $t+1$. As technologies diffuse their demand increases. Therefore, the market value of a technology must be indexed by its age. For a newly developed technology, age is 0, hence the value of the second subscript in $v_{t+1,0}^A$.

Free entry implies that the expected value of a new technology must equal its development cost:

$$W_t \frac{S_t^{1-\lambda_z}}{\zeta Z_{Dt}^{\phi} Z_{Wt}^{\xi} S_{Gt}^{\kappa}} = E_t\left[\Gamma_{t,t+1} v_{t+1,0}^A\right] \tag{19}$$

$$W_t S_t \left(Z_{Dt+1} - Z_{Dt}\right) = E_t\left[\Gamma_{t,t+1} v_{t+1,0}^A\right] \tag{20}$$

hence

$$S_t = \left[\frac{E_t\left[\Gamma_{t,t+1} v_{t+1,0}^A\right]}{Z_{Dt} g_{t+1} W_t}\right]^{\frac{1}{1-\lambda_z}} \tag{21}$$

To compute the market value of a newly developed technology, $v_{t+1,0}^A$, we make the simplifying assumption that after T periods (where T can be arbitrarily large but it is a fixed natural number), an intermediate good has fully diffused. After imposing this assumption, we compute $v_{t+1,0}^A$ by the following three-step procedure:

1. Let $\pi_{t,a}^x$ be the profits earned at time t from commercializing an intermediate good developed a periods ago. $\pi_{t,a}^x$ is equal to

$$\pi_{t,a}^x = \bar{\pi} x_t \Lambda_{t,a}$$

where

$$\begin{aligned} \bar{\pi} x_t &= (p_{it}^x - \bar{a} W_t) x_{ijt} = (\bar{\mu} - 1)\bar{a} Y_t W_t^{-\frac{1}{\theta-1}} \left(\frac{\alpha}{\bar{\mu}\bar{a}}\right) \Xi \\ &= (\bar{\mu} - 1) Y_t W_t^{-\frac{1}{\theta-1}} \left(\frac{\alpha}{\bar{\mu}}\right) \Xi \\ &= A^{-\mu} X_t (\bar{\mu} - 1)\bar{a} * W_t \end{aligned}$$

and $\Lambda_{t,a}$ is the share of producers that have adopted the intermediate good at t.

2. $\Lambda_{t,a}$ is computed as follows:

$$\Lambda_{t,a} = \begin{cases} 0 \text{ for } a = 0 \\ \Lambda_{t-1,a-1} + \lambda_{t-1}\left(1 - \Lambda_{t-1,a-1}\right) \text{ for } a \in (0, T) \\ 1 \text{ for } a = T \end{cases}$$

3. Compute $v_{t,0}^{A}$ recursively as

$$\begin{aligned} v_{t,0}^{A} &= \pi_{t,0}^{x} + E_t\left[\Gamma_{t,t+1} v_{t+1,1}^{A}\right] \\ v_{t,1}^{A} &= \pi_{t,1}^{x} + E_t\left[\Gamma_{t,t+1} v_{t+1,2}^{A}\right] \\ v_{t,2}^{A} &= \pi_{t,2}^{x} + E_t\left[\Gamma_{t,t+1} v_{t+1,3}^{A}\right] \\ &\ldots \\ v_{t,T}^{A} &= \pi_{t,T}^{x} + E_t\left[\Gamma_{t,t+1} v_{T,1}^{A}\right] \end{aligned}$$

Consumers

Consumers are standard. They choose the sequence of consumption and labor they supply to maximize the present discounted value of utility flows subject to the budget constraint and the transversality condition:

$$\underset{\{C_t,\, L_t,\, B_t\}_{t=0}^{\infty}}{Max} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \varsigma \frac{N_t^{1+\varphi}}{1+\varphi}\right]$$

s.t

$$B_{t+1} + C_t = W_t N_t + R_t B_t + \Pi_t \tag{22}$$

$$\lim_{t\to\infty} B_t \prod_{\tau=0}^{t} R_{\tau}^{-1} = 0 \tag{23}$$

where N_t denotes the hours worked per person, B_t denotes the holdings of a bond that has a zero aggregate net supply, and Π_t denotes the aggregate profits paid to consumers since they own all the corporations in the economy. The first order conditions of the consumer problem are:

$$\frac{W_t}{C_t} = \varsigma N_t^{\varphi} \tag{24}$$

$$E_t\left[\Gamma_{t,t+1} R_{t+1}\right] = 1 \tag{25}$$

where

$$\Gamma_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \tag{26}$$

Labor market equilibrium and aggregate resource constraint

The total number of hours supplied as labor by the representative consumer are used to produce intermediate goods (L_{xt}), produce final output (L_{yt}), conduct private (S_t) and public (S_{Gt}) R&D, and adopt new technologies (H_t).

$$N_t = L_{xt} + L_{yt} + S_t + S_{Gt} + H_t \tag{27}$$

Aggregate output is consumed.

$$Y_t = C_t$$

2.1.1 Discussion

Before moving on to the version of the model with Fraunhofer, it is worthwhile reflecting about some of the key features of the framework I have laid out so far, and its relationship to the literature. I will not devote time to discuss the semi-endogenous nature of the framework, as there is little novelty there. The novelty of the framework resides in the treatment of adoption and on the integration of a rich adoption setup into a model where innovations are fully endogenous.[4] Unlike most models of endogenous growth, in our setting firms must individually adopt the technologies they use. This creates a diffusion curve for each individual technology which reflects the fraction of firms that have successfully adopted a given technology at each point in time. Ours is one of the first endogenous growth models that incorporates this feature. I've already glossed how this feature adds realism to the model and hence precision to the quantification of the effects on the economy of any policy or institution. However, now I'd like to elaborate on the consequences that capturing the slow diffusion has for the structure of the model and, more precisely, for the model's complexity. Crucially, the slow diffusion of technologies implies that the demand faced by innovators depend on the innovation's age in addition to the time t. This feature makes the valuation of new technologies much more complex than in models without diffusion or with binary diffusion states. Specifically, $v^A_{t,0}$ cannot be defined recursively. To avoid the computational complexities of dealing with non-recursive Bellman equations, I impose the restriction that after T periods, where T is fixed and potentially large, the technology has fully diffused. This additional assumption has little quantitative bite, as T can be made arbitrarily large, and it allows me to develop an algorithm that computes the equilibrium of the economy.

2.2 Model with Fraunhofer

In addition to the combination of innovation and firm-level adoption, this framework is the first one that explicitly introduces Fraunhofer into an endogenous growth model and

[4]See Comin and Hobijn (2007) for a model where firms implement each technology determining the initial productivity it has. This investment does not affect the long-run productivity of the technology, just its initial level and the transition towards a pre-determined long-run productivity. This setting based on the implemntation of technologies differs from ours in that the implementation decision is made by the technology developer, not by the individual adopters.

analyzes the impact it has in the economy. Next I discuss how I capture Fraunhofer' activities in the model economy.

Fraunhofer conducts two key activities. First, it conducts applied research that expands the world technology frontier. Let $S_{Fraunhofert}$ denote the number of Fraunhofer scientists that work in applied R&D activities. For simplicity, we assume that Fraunhofer scientists in applied R&D activities are perfect substitutes with other public researchers. Formally, this means that the law of motion for domestically developed technologies is:

$$Z_{Dt+1} - Z_{Dt} = \zeta_{Dt} Z_{Dt}^{\phi} Z_{Wt}^{\xi} \left(S_{GDt} + S_{Fraunhofert}\right)^{\kappa} \; S_t^{\lambda_z} \tag{28}$$

The second relevant activity by Fraunhofer consists in bringing companies that engage in research contracts closer to the technology frontier. Note that, because Fraunhofer interacts only with a subset of the companies, in the model with Fraunhofer there is firm heterogeneity. In particular, there are two groups of firms, those that engage in research contracts and those that do not, with the former having higher adoption rates and number of adopted technologies. Formally, let $\bar{s}$ be the fraction of companies in the economy that engage in a research contracts with Fraunhofer. For those companies, the probability of adopting a new technology is:

$$\lambda_h \left(h_{jt}\right) = \lambda_0 \left(A_t h_{jt}\right)^{\rho_\lambda} + \lambda_F \tag{29}$$

Note that $\lambda_h(.)$ has two parts.[5] The first is the same as for firms that do not engage in research contracts and corresponds to the specification for the adoption rate in the model without Fraunhofer (equation 13). The second term, λ_F, reflects the contribution of Fraunhofer to the adoption rate of the firm. Note that, the marginal product of adoption workers in (29) is not affected by whether the firm is engaged in research contracts. Therefore, in the symmetric equilibrium, all firms in the economy will devote the same number of workers (h_t) to adopt each un-adopted technology.

The law of motion for the stock of adopted technologies in a firm that engages in research contracts (A_{ht})[6] is

$$A_{ht+1} - A_{ht} = \lambda_{ht} \left(Z_t - A_{ht}\right), \tag{30}$$

while for firms that do not engage in research contracts it is

$$A_{lt+1} - A_{lt} = \lambda_{lt}(Z_t - A_{lt}) \tag{31}$$

The average number of adopted technologies in the economy is

$$A_t = \bar{s} A_{ht} + (1 - \bar{s}) A_{lt}. \tag{32}$$

[5] In a slight abuse of notation, we denote by $\lambda_h(.)$ the adoption rate function of firms that engage in research contracts.

[6] I'm taking the liberty of starting to simplify notation anticipating the fact that in the symmetric equilibrium I analyze all firms of a given type are identical. As a result, I replace the j subindex that defines an individual firm, by the subindeces h and l that respectively denote a firm that engages (does not) in reserach contracts with FhG.

The profits earned by an intermediate good producer depend on the penetration of the intermediate good both among firms that engage in research contracts (Λ_{t,a_h}) and among firms that do not (Λ_{t,a_l}) as follows

$$\pi^x_{t,a} = \bar{\pi} x_t (\Lambda_{t,a\_h} \bar{s} + \Lambda_{t,a\_l}(1-\bar{s}))$$

Λ_{t,a_i} (for $i = \{h, l\}$) can be computed as follows:

$$\Lambda_{t,a\_i} = \begin{cases} 0 \text{ for } a = 0 \\ \Lambda_{t-1,a-1\_i} + \lambda_i \left(1 - \Lambda_{t-1,a-1\_i}\right) \text{ for } a \in (0, T) \\ 1 \text{ for } a = T \end{cases}$$

Labor market clearing condition now is

$$N_t = L_{xt} + L_{yt} + S_t + S_{Gt} + S_{Fraunhofert} + H_t + H_{Ft}$$

where H_{Ft} and $S_{Fraunhofert}$ are the total number of Fraunhofer workers involved, respectively, in research contracts and in applied R&D.

2.3 Equilibrium

A competitive symmetric equilibrium of the economy is defined as a sequence of quantities $\{Z_t, Z_{Dt}, Z_{Wt}, S_t, Y_t, Y_{ht}, Y_{lt}, A_{ht}, A_{lt}, X_{ht}, X_{lt}\ , x_{it}, x_{ijt}, L_{yt}, L_{xt}, H_t, h_t, \pi_{jt}, \left\{\Lambda_{t,a\_h}, \Lambda_{t,a\_l}, v^A_{t,a}\right\}^t_{a=0}\}^\infty_{t=0}$ and prices $\{W_t, p^x_{it}, P^X_{jt}, MC_{jt}, P^y_{jt}\}^\infty_{t=0}$ such that, given the exogenous variables $\{\zeta_{Dt}, \zeta_{Wt}, S_{Wt}, S_{GWt} S_{GDt}, S_{Fraunhofert}, H_{Ft}\}^\infty_{t=0}$, Z_{D0} and Z_{W0}, companies maximize profits, consumers maximize utility given the budget constraints and transversality conditions, and the labor market clears. In the version of the economy without Fraunhofer, $A_{ht} = A_{lt}$, $Y_{ht} = Y_{lt}$, and $\Lambda_{t,a\_h} = \Lambda_{t,a\_l}$.

Steady State

Our goal is to study the long-run impact of Fraunhofer on the German economy. Given this, it is natural to focus on the steady state of the economy. The steady state, is defined as an equilibrium of our model economy where all the variables grow at constant rates, possibly different across variables. In the Appendix we list the detrended variables and the equations that characterize the steady state of both versions of the model. Next we describe the growth rates of the relevant variables.

Growth rates

We denote the steady state growth rate of a generic variable M by g_M. In steady state A, Z , Z_D, Z_W all grow at constant rate $g = \frac{(\lambda_z+\kappa)n}{1-\phi-\xi}$. Output Y grows at rate

$$g_Y = (\theta - 1)g + n = \left[\frac{(\theta-1)(\lambda_z+\kappa)}{1-\phi-\xi} + 1\right] n$$

Labor productivity and the wage rate w grow at the rate:

$$g_{Y/L} = g_W = (\theta - 1)g$$

The growth rates of X, Q^A, q^A, h, v^A and V^A are

$$\begin{aligned} g_X &= (\mu - 1)g + n \\ g_{Q^A} &= g_Y \\ g_{q^A} &= g_Y - g = (\theta - 2)g + n \\ g_h &= \frac{(1+n)}{1+g} - 1 \simeq n - g \\ g_{v^A} &= n + g_{Y/L} - g = (\theta - 2)g + n \\ g_{V^A} &= n + g_{Y/L} = g_Y \end{aligned}$$

L_x, L_y, S, S_G, S_W, S_{GW}, H, grow at rate n. In the version with Fraunhofer, $S_{Fraunhofer}$, and H_F also grow at rate n.

3 Counterfactual exercise

The counterfactual exercise we conduct to assess the long-run contribution of Fraunhofer to the German economy consists in computing the steady state of the model economy with and without Fraunhofer and comparing the value of the relevant variables in the two steady states. To this end, we must first calibrate the model parameters. We do so by matching variables in the model with Fraunhofer to the relevant data counterparts in the German economy. To compute the equilibrium without Fraunhofer, we maintain fixed the values of the structural parameters, and set the values of $\bar{s}$, H_F and $S_{Fraunhofer}$ to zero. Next, I describe the calibration strategy and the results from the analysis.

3.1 Calibration

The Appendix contains a table with all the parameters and the values used in the baseline calibration. Next, I describe in detail how I set three broad classes of parameters. These are the parameters in the idea production function including relative productivities of R&D in Germany and abroad, the magnitude of Fraunhofer activities, and the direct impact that Fraunhofer has on the adoption rate for firms that engage in research contracts.

3.1.1 Idea production function parameters

The idea production function determines how R&D investments from the private and public sector lead to the creation of new technologies. To provide a full description of the idea production function, we need to characterize three types of parameters: the elasticities of R&D (λ_z and κ), the knowledge spillovers – domestic (ϕ) and international (ξ), and the productivity parameters (ζ_D, and ζ_W).

The approach followed to estimate the first two sets of parameters of the idea production function follows Bottazzi and Peri (2007). Bottazi and Peri develop a two stage approach to estimating all the elasticities in the idea production function. The first step uses the fact that the growth rate of the stock of patents (which is their proxy for Z, Z_D and Z_W) is stationary. Since the variables that appear in the right-hand-side of the idea production function are not stationary, they argue that there must exist a co-integrating relationship between them. Taking advantage of this observation, we can estimate $\kappa/(1-\phi)$, $\xi/(1-\phi)$ and $\lambda_z/(1-\phi)$. In a second stage, they pose an error correction model to estimate ϕ.[7]

The point estimates I use in the baseline calibration come from a recent study by Deloitte for the DG of Innovation at the European Commission. As Bottazzi and Peri (2007), Deloitte (2017) uses a cross-country panel of patent applications and R&D expenditures. The key difference (in addition to the time periods) is that while Bottazzi and Peri use patent applications from the USPTO, Deloitte mainly focuses on applications to the European patent office. The specific values I use are $\phi = .9$, $\xi = .08$, $\lambda_z = .06$, and $\kappa = (1-\lambda_z) * .06$. In the robustness check section, I consider Bottazzi and Peri estimates, as well as less conservative estimates from Deloitte for the domestic knowledge spillovers, ϕ.

In addition to the elasticities, we need to specify the R&D productivity parameters (ζ_D, and ζ_W). To this end, I proceed in two steps. First, I set the values of domestic public R&D labor ($S_{GD} + S_{Fraunhofer}$), foreign private and public R&D, to match the data.[8'9] I also set the markup ($\bar{\mu}$) so that aggregate private R&D matches the data. I also set the long-run ratio of domestic to foreign innovations to match the relative stock of patents in Germany and abroad from the European Patent Office. With this information, I compute the relative productivity of R&D by using the following expression which comes from imposing that in the long run both Z_D and Z_W growth at the same rate:

$$\frac{\zeta_D}{\zeta_W} = \left(\frac{Z_W}{Z_D}\right)^{\phi-1-\xi} \left(\frac{S_{GW}}{S_{GD} + S_{Fraunhofer}}\right)^{\kappa} \left(\frac{S_W}{S}\right)^{\lambda_z} \tag{33}$$

3.1.2 Magnitude of Fraunhofer activities

There are three exogenous variables that are critical for the counterfactual exercise: the share of Fraunhofer researchers involved in research contracts in total employment (H_F), the share of Fraunhofer researchers involved in R&D in total employment ($S_{g}\_Fraunhofer$), the share of firms engaged in research contracts ($\bar{s}$). We calibrate these variables using data from the OECD, and Fraunhofer's P&L accounts as follows:

$$H_F = \frac{\#Fraunhofer\text{ Researchers}}{\text{Total Emp. Ger.}} * \frac{\text{Rev. Research Contract (comp)}}{\text{Fraunhofer budget - Admin exp}} = .00006825,$$

[7] Intuitively, ϕ determines the rate of convergence to the cointegrating relationship after there is an exogenous deviation in one of the variables from the log-linear relationship that relates them in the long run.

[8] Note that these are exogenous variables, so I can fix them at any given value toe xplore the workings of teh model.

[9] Specifically, I normalize the detrended level of employment in germany in steady state to 1. Then I set S_{GD} and S_{FhG} to match the share of employment in public (non-FhG) and FhG R&D in total employment. Then I set S_{GW} and S_W to match the share of german's public and private R&D in the OECD plus China. In general, when I refer to the data in the paper I mean the average ratio since year 2000.

where $\frac{\#Fraunhofer \text{ Researchers}}{\text{Total Emp. Ger.}}$ is the share of Fraunhofer researchers to total employment in Germany, and $\frac{\text{Rev. Research Contract (comp)}}{\text{Fraunhofer budget - Admin exp}}$ is the ratio of Fraunhofer revenues from private research contracts to total revenues net of administrative costs.

$$S_g\_Fraunhofer = \frac{\#\text{ Public researchers}}{\text{Total Emp. Ger.}} \frac{\text{Fraunhofer Rev.} * (\text{non-contract Fraunhofer Revenues })}{\text{Public R\&D Exp}} = .0002,$$

where $\frac{\text{Fraunhofer Rev.}*(\text{non-contract Fraunhofer Revenues })}{\text{Public R\&D Exp}}$ is the ratio of Fraunhofer non-research revenues to total public R&D expenditures in Germany.

$$\overline{s} = \frac{\overbrace{\frac{\text{Total Exp in Research Contracts}}{\text{Avg. Exp per research contract}}}^{\#\text{ Companies involved with Fraunhofer}}}{\underbrace{\frac{\text{Private GDP}}{\text{Avg. Value added of companies in research contract}}}_{\#\text{ Companies in Germany}}} = .0031,$$

where $\frac{\text{Total Exp in Research Contracts}}{\text{Avg. Exp per research contract}}$ is the equivalent number of companies involved in Fraunhofer research contracts (where equivalent refers to the average size of research contracts), and $\frac{\text{Private GDP}}{\text{Avg. Value added of companies in research contract}}$ is the average number of equivalent firms in Germany computed as the ratio of private GDP to the average value added of firms engaged in Fraunhofer research contracts.

To put this figures in perspective, total Fraunhofer R&D expenditures (including applied R&D and the expenditures involved in completing research contracts) represent 0.061% of Germany's GDP, 2.37% of Germany's R&D expenditures, and 7.67% of public German R&D.

3.1.3 Direct impact of research contracts

Without a doubt, the most complex parameter to calibrate is how much do Fraunhofer research contracts help companies close the gap with the technology frontier, which in the model is captured by λ_F. To calibrate this parameter entails significant complexities. Fortunately, I can build on very pertinent prior work that, together with the model, can help me calibrate precisely the parameter λ_F. Specifically, Comin et al. (2019) estimate the effect that engaging with Fraunhofer though research contracts has on firm-level value added.

To this end, we assemble a firm-level data set that contains the universe of firms that have signed research contracts as well as firm performance measures. A key challenge to identifying the causal effect of Fraunhofer on firm performance is that firms may self-select into contracting with Fraunhofer. If firms with higher value added are more likely to engage with Fraunhofer, standard econometric techniques may result in biased estimates of the effect of research contracts on company's performance. To deal with the potential endogeneity of the firms' interactions with Fraunhofer, we take advantage of the presence of cross-sectional scale-heteroscedasticity in the selection equation, and construct the sort of instrumental variables described in Lewbel (2012).[10] Using a dynamic specification where we allow for the possibility that current interactions with Fraunhofer also increase the chance of future interactions, we estimate that, the cumulative engagement with Fraunhofer leads the value added of the firm to an asymptotic increase of 20.5%.

In addition to this micro estimate, pinning down the value of λ_F requires a mapping between firm-level sales and the adoption rate. Our model provides this mapping, and allows us to compute by how much research contracts affect adoption rates.[11] The resulting value for λ_F is 0.144.

	With Fraunhofer	Without FhG	Only FhG R&D	Only FhG Res. Contracts
Y	.9096	.8764	.9092	.8769
N	1	.9999	1	.9999
S	.006	.0058	.006	.0058
Z	1	.8655	.9993	.8661
A	.6827	.5906	.6818	.5917
Y/L	.9096	.8765	.9092	.877
W	.7644	.7364	.7641	.7367
Adopt. Rate	.1254	.125	.125	.1259

Table 1, Counterfactual Simulations, levels

3.2 Results

After these preliminaries, we can now use the calibrated models with and without Fraunhofer to study the long-run effect of Fraunhofer on the German economies. Table 1 presents the values of the variables of interest in four different scenarios. The first column presents the values in the version of the model with Fraunhofer. The second column, presents the values in the version of the model without Fraunhofer. That is, if we assume that Fraunhofer does not conduct public R&D (i.e., $S_{Fraunhofer} = 0$), and there are no research contracts ($H_F = \bar{s} = 0$). Note additionally that, since we keep S_G constant, we are assuming that no other public organization is replacing the void left by Fraunhofer.[12] Column 3 and Column 4 report the results from eliminating only one of the two activities that Fraunhofer conducts. In Column 3, we eliminate research contracts (and keep constant the value of $S_{Fraunhofer}$), and in column 4 we eliminate the public R&D portion of Fraunhofer and keep research

[10] The key identification assumption we rely on is that the variance of the unobserved variables that may affect firm performance is not correlated to the variables that drive heteroscedasticity in first stage, which in our case is firm employment. If this condition holds, our instrument is exogenous and therefore valid.

[11] Intuitively, value added depends on th firm level of adopted technologies, which, in steady state, is determined by λ_i. Since the gap between the adoption rate of firms witha nd without research contracts is λ_F, we can readily map the estimate of teh long-run contribution of FhG to the firm's value added to λ_F.

[12] At this point, it is relevant to make a side comment on the fact that, in this study, I have not tried to study the productivity of different types of public R&D, and how substitutable they are. In this regards, the reduction of public R&D in the equilibrium without FhG, other things equal, increases the amount of labor available to conduct other activities. I have not considered, however, the possibility that other public institutions increase their R&D so that $S_G + S_{FhG}$ remains unaffected.

contracts as in the equilibrium with Fraunhofer. Table 2 reports the percentage change relative to the steady state values in the version of the model with Fraunhofer (column 1).

Eliminating Fraunhofer reduces Germany's long-run GDP level by 3.65%. This the case because by eliminating Fraunhofer, private R&D in Germany declines by 3.33%. As a result, the level of technology in the frontier (Z) declines by 13.45%, and the long-run level of adopted technologies declines by 13.49%. The reduction in the stock of adopted technologies leads to a reduction in labor productivity and wages by 3.64% and 3.66%, respectively. Hours worked decline by 0.01%.

	% Decline with FhG		
	Without FhG	Only FhG R&D	Only FhG Res. Contracts
Y	3.65	.04	3.59
N	.01	0	.01
S	3.33	0	3.33
Z	13.45	0.07	13.39
A	13.49	0.13	13.33
Y/L	3.64	0.04	3.58
W	3.66	0.04	3.62
Adopt. Rate	.36	.36	-.4

Table 2: Counterfactual simulations, percent decline from baseline economy with Fraunhofer.

3.2.1 Understanding the channels

What are the channels driving the impact of Fraunhofer in the economy? In particular, is the decline in innovation and adoption activities driven by Fraunhofer's contribution to public R&D or by the effect of research contracts?

Comparing the simulations reported in columns 3 and 4 of Table 2, we can conclude that the public R&D channel is much more important than the research contracts channel. This is the case because eliminating Fraunhofer leads to a reduction in public R&D by 7.67% which causes a very significant reduction in private R&D and consequently in the long-run stock of technologies in the economy.

Eliminating research contracts reduces the speed of adoption lowering the level of adopted technologies relative to the frontier. This effect seems minor relative to Fraunhofer's impact through public R&D. However, it is very significant relative to the budget of research contracts. Our simulations show that research contracts increase long-run GDP by 0.05%. Their cost is 0.006% of GDP. Therefore, the social value of research contracts is 9 time larger than their cost.

3.3 Robustness

To demonstrate the robustness of our conclusions, we recompute our calculations using reasonable variations of the key parameters, especially those for which we have greater uncertainty about their values. Broadly speaking, the checks I conduct fall under two categories. The first corresponds to variations in the values used for the parameters in the idea production functions (rows 2, 4 and 6 of Table 3). The second correspond to variations in the calibration of the adoption function (rows 3 and 5 of Table 3). The first row of Table 3 reports the baseline values, for comparison purposes. For concreteness, I focus on the % decline in the detrended steady state level of GDP when Fraunhofer is completely eliminated.[13]

Specifically, in row 2 of Table 3, I increase quite significantly κ (the elasticity of new ideas with respect to public R&D) to make it consistent with the estimates in Deloitte (2017). In row 5, I increase the size of the domestic knowledge spillovers to match the estimates in Deloitte (2017). In row 6, I use the values of the international knowledge spillovers (ξ) and private R&D elasticity (λ_z) estimated in Bottazzi and Peri (2007). In row 3, I increase the productivity of adoption investments so that the equilibrium adoption rate for firms without research contracts increases by 20%. In row 5, I increase the elasticity of the adoption rate with respect to adoption expenditures. Importantly, in all these exercises, we keep constant the steady state growth rates of population, and more importantly of output. Note that, because the growth rate of ideas depends on the parameters of the idea production function, the growth rate of technology will vary across the different simulations. To keep constant the steady state growth rate of output, we adjust the elasticity θ so that when the growth rate of ideas declines, the increase in θ maintains constant the growth rate of output.

The main take away from our simulations is that the magnitude of the long-run impact that Fraunhofer has on the German economy is very robust. In particular, in only one of the alternative parameterizations there is a mild reduction in the effect of Fraunhofer (from 3.65% of GDP in the baseline to 3.44% in the Bottazzi and Peri parameterization). Intuitively, in the Botazzi and Peri simulation, the production of new ideas is more elastic with respect to private R&D but less elastic with respect to the international stock of ideas, and public R&D. What our simulations demonstrate is that under this new parameterization, the latter forces dominate. In particular, the decline in long-run productivity due to the lower public R&D in the equilibrium without Fraunhofer is smaller when the elasticity of ideas with respect to public R&D is smaller. This is the case despite the higher elasticity of ideas with respect to private R&D that Bottazzi and Peri estimate.

	ϕ	ξ	λ_z	κ	ρ_λ	% GDP Decline without FhG
Baseline	.9	.08	.06	.0564	.7	3.65
High κ	.9	.08	.06	**.282**	.7	6.07
High λ_l	.9	.08	.06	.0564	.7	3.66
High ρ_λ	.9	.08	.06	.0564	.95	3.64
High ϕ	**.95**	.08	.03	.0582	.7	4.91
Bottazzi and Peri	.9	**.056**	**.08**	.0552	.7	3.44

Table 3: Robustness Checks

[13] Additionally, in simulations not reported in the table, I have check the sensitivity of the results to the calibration of markups and elasticities of substitution. The results are completely robust with regards to broad variation in the values of those parameters.

In one case, we find that the impact of Fraunhofer is much greater than in the baseline. That is when we increase the elasticity of ideas with respect to public R&D. This should not be surprising in the light of the robustness check in row 6, and in the light of the analysis of the relative contribution of Fraunhofer via research contracts vs. public R&D.

4 Conclusions

In this paper, I have developed a general equilibrium model with endogenous development and adoption of technologies at the firm level. The model has been augmented to recognize the dual role of Fraunhofer as an institution that conducts public R&D and that helps some firms close their technological gap with the frontier through research contracts. To calibrate the model, I have used two prior studies that provide direct information necessary to set the parameter values in the ideas production function and on the effectiveness of research contracts. These studies were conducted long before I was commissioned this paper. I have used the calibrated model to calculate the long-run contribution of Fraunhofer to the German economy.

The key finding is that Fraunhofer leads to a long-run level of GDP in Germany 3.65% higher than if Fraunhofer did not existed. The magnitude of this effect is robust. Reasonable variations in the key parameter values show that if anything the estimated impact of Fraunhofer is conservative.

Both the public R&D and research contracts contribute to this effect, though the effect of Fraunhofer research on GDP via the increase in public R&D is much greater than the effect of research contracts on technology adoption. This finding should not mask the fact that the social return to Fraunhofer research contracts is huge, with an impact on GDP 9 time greater than its cost.

These conclusions are the direct outcome from the analyses I have conducted in this paper. The framework I have developed, its calibration and simulation are done with complete objectivity and in no way they are influenced by the fact that this study is commissioned and compensated by Fraunhofer.

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A Steady states

In this section, I characterize the stationary variables that define the steady states of the model with and without Fraunhofer.

A.1 Steady state with Fraunhofer

Without loss of generality, we normalize the asymptotic detrended levels of Z, and population to 1. Given the exogenous variables: $S^G, S_{Fraunhofer}, H_F, S^G_W, S_W$, the steady state of the economy (with Fraunhofer) is defined by the following variables 26+4*T variables: $A_h, A_l, A, \bar{Z}_W, \bar{Z}_D, W, Q_{A}\_W = Q_A/W, Y, y_h.y_l, L_x, L_{xh}.L_{xl}, L_y, L_{yh}.L_{yl}, X_h, X_l, H_z, H, \lambda_h, \lambda_l, R, C, N, S,$ $\left\{\Pi^x_a\_W, V^A_a\_W, \Lambda_{a\_h}, \Lambda_{a\_l}\right\}^T_{a=0}$,which are pinned down by the following 26+4*T equations:

Production:

$$Y = \left[\bar{s}\, y_h^{1/\theta} + (1-\bar{s}) y_l^{1/\theta}\right]^\theta \tag{34}$$

$$y_h = A_h^{\phi-\alpha(\mu-1)} X_h^\alpha L_{yh}^{1-\alpha} \tag{35}$$

$$y_l = y_h * (A_l/A_h)^\theta \tag{36}$$

$$X_h = A_h^\mu Y w^{-\frac{\theta}{\theta-1}} \left(\frac{\alpha}{\bar{\mu}\bar{a}}\right) \Xi \tag{37}$$

$$X_l = X_h * (A_l/A_h)^\mu \tag{38}$$

$$L_{yh} = (1-\alpha) Y w^{-\frac{\theta}{\theta-1}} A_h \Xi \tag{39}$$

$$L_{yl} = L_{yh} * (A_l/A_h) \tag{40}$$

$$L_y = \bar{s} * L_{yh} + (1-\bar{s}) * L_{yl} \tag{41}$$

$$L_{xh} = Y_t w_t^{-\frac{\theta}{\theta-1}} \left(\frac{\alpha}{\bar{\mu}}\right) A_h \Xi \tag{42}$$

$$L_{xl} = L_{xh} * (A_l/A_h) \tag{43}$$

$$L_x = \bar{s} * L_{xh} + (1-\bar{s}) * L_{xl} \tag{44}$$

Technology:

$$A_h = \left[1 + \frac{g}{\lambda_h}\right]^{-1}$$

$$A_l = \left[1 + \frac{g}{\lambda_l}\right]^{-1}$$

$$A = \bar{s} A_h + (1-\bar{s}) A_l$$

$$\frac{\bar{Z}_D}{\bar{Z}_W} = \left[\frac{\zeta}{\zeta_W} \left(\frac{S^G}{S^G_W}\right)^\kappa \left(\frac{S}{S_W}\right)^\lambda\right]^{\frac{1}{1-\phi+\xi}}$$

$$\bar{Z}_D + \bar{Z}_W = 1$$

Value of upgrading technologies and optimal upgrading:

Let $\Xi_W = \left[\frac{(1-\alpha)^{1-\alpha}}{\bar{\theta}\left(\frac{\bar{\mu}\bar{a}}{\alpha}\right)^{\alpha}}\right]$, $\Xi = \frac{\Xi_W^{\frac{1}{\theta-1}}}{\bar{\theta}}$.

$$\begin{aligned}
W &= A^{\theta-1}\Xi_W \\
Q_{A}_W &= \frac{YA^{-(\theta-1)}\left(1-1/\bar{\theta}\right)}{\Xi_W\left[1-\frac{(1+g_Y)}{R(1+g)}\right]} \\
H_z &= Q_{A}_W\frac{\rho\lambda_l Z}{A}\frac{(1+g_Y)}{R(1+g)} \\
H &= H_z\left[1-(\bar{s}\frac{A_h}{Z}+(1-\bar{s})\frac{A_l}{Z})\right]
\end{aligned}$$

Diffusion: For $i = \{h, l\}$,

$$\Lambda_{a\_i} = \begin{cases} 0 \text{ for } a = 0 \\ \Lambda_{a-1\_i} + \lambda_{\_i}\left(1-\Lambda_{a-1\_i}\right) \text{ for } a \in (0, T) \\ 1 \text{ for } a = T \end{cases}$$

where

$$\begin{aligned}
\lambda_l &= \lambda_0\left(A * H_z\right)^{\rho_\lambda} \\
\lambda_h &= \lambda_l + \lambda_F
\end{aligned}$$

Value of new technologies and free entry:

$$\Pi^x_{a}\_W = \frac{(\bar{\mu}-1)\alpha}{\bar{\mu}\bar{\theta}\Xi_W}YA^{-(\theta-1)}(\Lambda_{a\_h}\bar{s} + \Lambda_{a\_l}(1-\bar{s}))$$

$$\begin{aligned}
V^A_0_W &= \Pi^x_0_W + \left[\frac{V^A_1_W(1+g_Y)}{R(1+g)}\right] \\
V^A_1_W &= \Pi^x_1_W + \left[\frac{V^A_2_W(1+g_Y)}{R(1+g)}\right] \\
V^A_2_W &= \Pi^x_2_W + \left[\frac{V^A_3_W(1+g_Y)}{R(1+g)}\right] \\
&\dots \\
V^A_T_W\left(1-\frac{(1+g_Y)}{R(1+g)}\right) &= \Pi^x_T_W \\
S &= \frac{V^A_0_W}{R}\frac{g * Z_D * (1+gY)}{A(1+g)}
\end{aligned}$$

Market clearing

$$N = L_x + L_y + S + S_G + S_{Fraunhofer} + H + H_F$$

Consumer choices and resource constraint

$$\frac{W}{C} = \varsigma N^{\varphi} \tag{45}$$

$$R = (1 + g_{Y/L})/\beta$$

$$Y = C \tag{46}$$

A.2 Steady state without Fraunhofer.

The economy without Fraunhofer is defined by the previous system after imposing the following constraints: $S_{Fraunhofer} = H_F = \bar{s} = 0$, $A_h = A_l$, $L_{yh} = L_{yl}$, $L_{xh} = L_{xl}$, $X_h = X_l$ $y_h = y_l$, $\lambda_h = \lambda_l$, $\Lambda_{a\_h} = \Lambda_{a\_l}$.

B Calibration

The following table contains the values used to calibrate all the parameters in the baseline model.

$\bar{\mu}$	1.2642	n	.01
β	.96	κ	0.0564
ς	0.8404	ξ	0.08
$\bar{a}$	.1	ϕ	.9
φ	2	λ_z	.06
$\bar{s}$	0.0031	λ_F	.1441
H_F	0.00006825	S_{GD}	0.0024
λ_0	0.3524	$S_{Fraunhofer}$	0.000170659
ρ_λ	.7	S_D/S_W	0.06
α	1/3	$(S_{GD} + S_{Fraunhofer})/S_{GW}$	0.05
μ	1.3	Z_D/Z_W	.2
θ	1.2577	ζ_D/ζ_W	1.0492
$\bar{\theta}$	1.25		